5th International Conference
on High Performance Marine Vehicles,
8-10 November, 2006, Australia

Investigation of Sway, Roll and Yaw Motions of a Ship with Forward Speed: Numerical modeling for flared up conditions

S.K. Das, International Institute of Information Technology, Pune, India, samird@isquareit.ac.in
S.N. Das, Central Water & Power Research Station, Pune, India, sndas_cwprs@rediffmail.com
P.K. Sahoo, Australian Maritime College, Launceston, Australia, P.Sahoo@mte.amc.edu.au

Abstract

The paper deals with the numerical modeling of sway, roll and yaw motions of a ship for flared up conditions with zero or nonzero forward speeds in sinusoidal waterway. To compute hydrodynamic forces, we employ nonlinear roll restoring characteristics and speed dependent strip theory that are obtained from the Frank's close-fit method. The governing equations are solved numerically by using Runge-Kutta-Gill method with adaptive step size adjustment algorithm. In order to investigate the effect of nonlinear restoring in roll, numerical experiments have been carried out for a Panamax Container ship under the action of sinusoidal wave of periodicity 11.2 sec with varying wave height and speed. To emulate the soft spring behaviour, nonlinear restoring moment is represented by an odd order polynomial of roll angle where the corresponding coefficients are obtained by analyzing the results of numerical experiments. This modeling approach provides an important guideline to understand the role of various parameters while flared up conditions does occur together with its controlling mechanisms.

Keywords

Sway, Roll, Yaw, Numerical modeling, Flared up motions, Froude-Krylov force.

1. Introduction

In ship motion studies, the analysis of large amplitude nonlinear rolling is important to understand capsizes dynamics. Quite often, the motion responses are flared-up to an extent when roll-restoring moment poses serious stability problem while the ship moves with forward speed. For such roll analysis, linear approximation is no longer valid (Bhattacharyya, 1978) and as a result, obtaining closed form solution becomes difficult.

In the past, several researchers Haddara (1973), Roberts (1984), Cardo et al.,(1984), Nayfeh and Khdeir (1986) and Virgin (1987) have analyzed the effects of nonlinear restoring moments of a rolling ship. Haddara (1973) developed an analytical method to obtain an approximate solution corresponding to nonlinear rolling equation of a ship in random waves. Virgin (1987) and Cardo et al. (1984) have examined the influence of nonlinear ship rolling in regular seas by applying Poincare mapping techniques to include chaotic motion and perturbation analysis respectively. Nayfeh and Khdeir (1986) obtained a second order approximate solution for nonlinear harmonic roll motion using perturbation analysis as well as numerical method to obtain limit cycles. Roberts (1984) estimated the roll response process by making comparison between simulation results and theoretical predictions. However, most of the earlier analyses were restricted to study of uncoupled rolling in beam waves.

In this paper, we examine the behaviour of nonlinear roll restoring for coupled system (sway-roll-yaw) of a ship, moving with constant forward speed in sinusoidal waves. To simulate the soft spring action, linear, cubic and quintuple dependence of roll angle is considered on extending the mathematical modeling approach given by Das and Das (2004) for a stationary ship. Using the strip theory approach of Salvesen et al., (1970) the sectional coefficients are integrated along the longitudinal axis of the ship by applying Frank and Salvesen's close fit method (1970) based on the experimental results of Vugts (1968). To obtain the realization of roll responses in coupled conditions, governing equations are solved numerically with the variation of ship speed. This enables us to examine the sensitivity of initial conditions and flared-up conditions for a container ship.

2. Problem formulation

A cartesian co-ordinate system (x,y,z) fixed with respect to the mean position of the ship is considered with z-axis acting in the vertical upward direction and the origin O lies in the undisturbed free surface. It is assumed that the ship is a rigid and slender body symmetric about x-z plane, and the centre of gravity G is located at (z_0 , o, z_c), where z_0 = OO' and z_c = O'G (Figure 1).

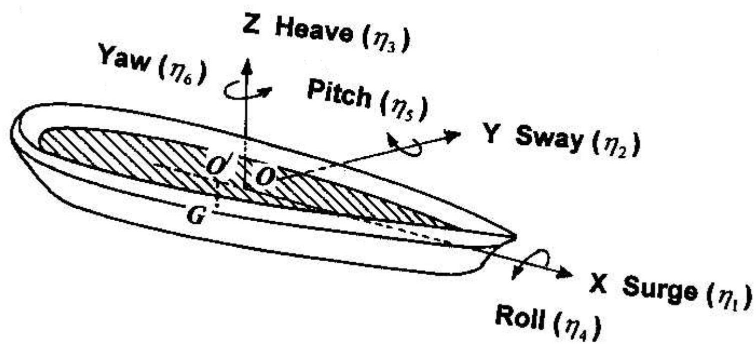

Figure 1: Motion and Coordinate System definition of a floating body

The ship is excited by monochromatic waves of frequency ω, and the force components generated by the propeller and wind are neglected. The translatory displacements along the x, y and z directions can be described kinematically in terms of surge (η_1), sway (η_2) and heave (η_3), and the angular displacements of the rotational motion about the same set of axes are roll (η_4), pitch (η_5) and yaw (η_6). In ship motion studies, frequency response analysis corresponding to a Fourier approach can be conveniently applied, Tick (1959). Owing to complex interactions between the hull and ship generated waves; the governing equations can be written in the form of integro-differential equations, which poses enormous difficulty in solving, Cummins (1962). Such difficulty can be conveniently avoided by considering the occurrence of ship motion under the action of regular waves. This reduces the integro-differential equation into ordinary differential equation (ODE). Following the approach of Tick (1959) for coupled system in three-degrees of freedom, sway-roll-yaw motions can be written as:

$$\sum_{k=2,4,6}\left[-\omega^2 \mathrm{M}_{jk}(\omega)\,\mathrm{X}_k(\omega)\,e^{i\omega t}+i\omega \mathrm{B}_{jk}(\omega)\mathrm{X}_k(\omega)\,e^{i\omega t}+C_{jk}(\omega)\mathrm{X}_k(\omega)\,e^{i\omega t}\right]$$
$$=D_j(\omega)F_j(\omega)e^{i\omega t}$$
$$j=2,4,6 \qquad (1)$$

where $\mathrm{X}_k(\omega)$ is the displacement, $F_j(\omega)$ is the wave force with amplitude $D_j(\omega)$; $\mathrm{M}_{jk}(\omega)$,

$\mathrm{B}_{jk}(\omega)$ and $C_{jk}(\omega)$ are the virtual mass, damping and restoring moments corresponding to the wave frequency ω respectively. Now, defining

$$\eta_k(t) = \mathrm{X}_k(\omega)e^{i\omega t},\ f_k(t) = F_k(\omega)e^{i\omega t},\ k = 2,4,6 \tag{2}$$

one can obtain

$$\sum_{k=2,4,6}\left[\mathrm{M}_{jk}(\omega)\ddot{\eta}_k(\mathrm{t}) + \mathrm{B}_{jk}(\omega)\dot{\eta}_k(\mathrm{t}) + C_{jk}(\omega)\eta_k(\mathrm{t})\right] = D_j(\omega)f_j(t),\ j = 2,4,6 \tag{3}$$

It is apparent that the motion variables (η_i), exciting force $f_j(t)$ and wave frequency (ω) described in equation (3) are complex quantities, and these can be expressed as algebraic sum of real and imaginary parts. Accordingly, the forcing function $f_j(t)$ becomes

$$f_j(t) = F_j(\omega)e^{i(\omega_R + i\omega_I)t} = F_j(\omega)e^{i\omega_R t}e^{-\omega_I t} \tag{4}$$

For simplicity, we assume the imaginary part of wave frequency (ω_I) to be equal to zero, yielding

$$f_j(t) = F_j(\omega)e^{i\omega_R t} \tag{5}$$

The motion responses and forcing functions can also considered as sum of real and imaginary parts:

$$\eta_j = \eta_{jR} + i\eta_{jI} \text{ and } F_j = F_{jR} + iF_{jI},\quad \mathrm{j} = 2,4,6 \tag{6}$$

Considering only the real part of motion response and exciting moment for a given wave frequency, the equation of motion for coupled sway-roll-yaw can be described as, Hooft (1982):

$$[d_{i2}\eta_2(t) + d_{i4}\eta_4(t) + d_{i6}\eta_6(t)] = F_i(t),\ \mathrm{i} = 2,4,6 \tag{7}$$

where the operator d_{ij} is given by

$$d_{ij} = \Delta_{ij}(t)\frac{d^2}{dt^2} + B_{ij}(t)\frac{d}{dt} + C_{ij}(t),\qquad \mathrm{i, j} = 2,4,6 \tag{8}$$

$F_i(t)$, i = 2,4,6 is the wave exciting force or moment, $\Delta_{ij}(t) = M_{ij} + A_{ij}(t)$ is the virtual mass moment of inertia, $A_{ij}(t)$, $B_{ij}(t)$ and $C_{ij}(t)$ are the cross-coupled coefficients like added mass, damping and restoring in the direction i due to any motion in the direction j. Using equations (7) and (8), the governing equations can be expressed in the following matrix form:

$$\left\{[M_{ij}] + [A_{ij}]\right\}[\ddot{\eta}_i] + [B_{ij}][\dot{\eta}_i] + [C_{ij}(t)][\eta_i] = [F_j(t)],\quad \mathrm{i,j} = 2,4,6 \tag{9}$$

The coefficient matrices can be expressed as

$$\lfloor M_{ij} \rfloor = \begin{bmatrix} M & -Mz_C & 0 \\ -Mz_C & I_4 & -I_{46} \\ 0 & -I_{64} & I_6 \end{bmatrix}, \lfloor A_{ij} \rfloor = \begin{bmatrix} A_{22} & A_{24} & A_{26} \\ A_{42} & A_{44} & A_{46} \\ A_{62} & A_{64} & A_{66} \end{bmatrix},$$

$$\left[B_{ij}\right]=\begin{bmatrix} B_{22} & B_{24} & B_{26} \\ B_{42} & B_{44} & B_{46} \\ B_{62} & B_{64} & B_{66} \end{bmatrix},\ \left[C_{ij}(t)\right]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{44}^{r}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix},\ [\ddot{\eta}_i(t)]=\begin{bmatrix} \ddot{\eta}_2(t) \\ \ddot{\eta}_4(t) \\ \ddot{\eta}_6(t) \end{bmatrix},\ [\dot{\eta}_i(t)]=\begin{bmatrix} \dot{\eta}_2(t) \\ \dot{\eta}_4(t) \\ \dot{\eta}_6(t) \end{bmatrix},$$

$$[\eta_i(t)]=\begin{bmatrix} \eta_2(t) \\ \eta_4(t) \\ \eta_6(t) \end{bmatrix},\ [F_j(t)]=\begin{bmatrix} F_2(t) \\ F_4(t) \\ F_6(t) \end{bmatrix} \tag{10}$$

where M is the mass of the ship, and the components in the matrices $[\dot{\eta}_i(t)]$ and $[\ddot{\eta}_i(t)]$ indicate time derivatives. $F_2(t)$, $F_4(t)$ and $F_6(t)$ are the wave forces or moments for sway, roll and yaw. I_j is the moment of inertia in the j^{th} mode, and I_{jk} is the product of inertia. In this formulation, the added mass and damping coefficients are frequency dependent, however, can account speed-dependent variations. These are computed by integrating two-dimensional sectional coefficients corresponding to known wave frequency along the length of the body, Salvesen et al., (1970). In the present study, we investigate flared up conditions when the restoring moment is nonlinear. Here, we consider the restoring moment $C_{44}^{r}(t)$ is having functional dependence on roll angle η_4, and can be expressed as an odd order polynomial of η_4 , [(Dalzell (1978), Bhattacharryya (1978), Cardo et al.,(1980) and Nayfeh and Khdeir (1986)].

$$C_{44}^{r}(t)=\Delta\overline{G}\overline{Z}(\eta_4)=C_{44}^{1}\eta_4+C_{44}^{3}\eta_4^{3}+C_{44}^{5}\eta_4^{5} \tag{11}$$

where Δ is the displacement in weight, $\overline{G}\overline{Z}(\eta_4)$ is the righting arm, C_{44}^{1}, C_{44}^{3} and C_{44}^{5} are coefficients. The equation (11) represents nonlinear restoring, and can emulate the behaviour of soft spring action. To represent linear roll restoring, one may consider C_{44}^{3} and C_{44}^{5} as zero, yielding

$$C_{44}^{r}(t)=\Delta\overline{G}\overline{Z}(\eta_4)=C_{44}^{1}\eta_4 \tag{12}$$

where $C_{44}^{1}=\rho\, g\nabla\,\overline{G}\overline{M}$ (13)

Here ∇ is the displaced volume, $\overline{G}\overline{M}$ is the metacentric height, ρ is the density of water and g is the acceleration due to gravity.

The wave exciting forces and moments are expressed in sinusoidal form as

$$F_i(t)=F_i^{A}\sin(\omega t+\theta\ ),\quad i=2,4,6 \tag{14}$$

where F_2^{A}, F_4^{A} and F_6^{A} are the amplitudes of the sway exciting force, roll exciting moment and yaw exciting moment respectively, θ is the phase angle, and ω is the encountering wave frequency. The amplitudes of the sway exciting force, roll exciting moment and yaw exciting moment can be obtained as per Salvesen et al., (1970),

$$F_2^{A}=\alpha\,\rho\int(f_2+h_2)d\xi \tag{15}$$
$$F_4^{A}=\alpha\,\rho\int(f_4+h_4)d\xi \tag{16}$$

$$F_6^A = \alpha \rho \int \xi \ (f_6 + h_6) d\xi \tag{17}$$

where α is the amplitude of the incident wave, f_i and h_i represent the 2D sectional Froude-Krylov force and diffraction force respectively.

3. Modelling for nonlinear restoring and experiment

The governing equation (9) comprised of sway, roll and yaw equations is analytically intractable owing to the presence of nonlinear roll restoring term $C_{44}^r(t)$. In the absence of nonlinear term, the governing equations can be solved analytically. The detailed description of the analytical method by considering linear damping and linear restoring moment for two and three degrees of freedom and without considering speed dependent sectional coefficients can be obtained from the investigations of Das and Das (2005,2004,2006a). However, for uncoupled roll with nonlinear added mass and damping, Das et al., (2006b) have obtained numerical solution when the ship is either stationary or moving with a forward speed of 10 knots. In the present study, the coupled sway-roll-yaw governing equations are solved numerically to get the effect of nonlinear roll restoring on other motions. In order to solve these equations, numerical integration based on Runge-Kutta method has been adopted, Press et al., (1992). In this case, three-second order ordinary differential equations (9) are transformed into six first order ordinary differential equations, assigning appropriate initial conditions:

$$\dot{\eta}_2(t) = \phi_2 \tag{18}$$

$$\dot{\eta}_4(t) = \phi_4 \tag{19}$$

$$\dot{\eta}_6(t) = \phi_6 \tag{20}$$

$$\dot{\phi}_2 = \ddot{\eta}_2(t) = \begin{bmatrix} F_2(t) - B_{22}\,\phi_2 - (A_{24} - \mathrm{M}\ z_C)\dot{\phi}_4 - B_{24}\phi_4 \\ - A_{26}\dot{\phi}_6 - B_{26}\phi_6 \end{bmatrix} / (A_{22} + M) \tag{21}$$

$$\begin{aligned}\dot{\phi}_4 = \ddot{\eta}_4(t) = [&F_4(t) - (A_{42} - Mz_C)\dot{\phi}_2 - B_{42}\phi_2 - B_{44}\phi_4 - C_{44}^1\eta_4 - C_{44}^3\eta_4^3 - C_{44}^5\eta_4^5 \\ &- (A_{46} - I_{46})\dot{\phi}_6 - B_{46}\phi_6] / \left(A_{44} + I_4\right)\end{aligned} \tag{22}$$

$$\dot{\phi}_6 = \ddot{\eta}_6(t) = \begin{bmatrix} F_6(t) - A_{62}\dot{\phi}_2 - B_{62}\phi_2 - (A_{64} - I_{46})\dot{\phi}_4 \\ - B_{64}\phi_4 - B_{66}\phi_6 \end{bmatrix} / \left(A_{66} + I_6\right) \tag{23}$$

The system of equations (18) to (23) with prescribed conditions poses well-defined initial value problem, which are being solved through step-by-step integration procedure. As the roll motion is coupled with sway and yaw, implicit dependence of these motions on nonlinear restoring is explored with the variation of initial condition, wave height and ship's speed. The variation of ship's speed is also accounted in the formulation to examine the rolling behaviour. Applying Runge-Kutta-Gill method with step-size adjustment algorithm, desired accuracy is achieved. The righting-arm curve or the $\overline{G}\overline{Z}$ curve is represented here by a fifth order polynomial;

$$\overline{G}\overline{Z}(\eta_4) = \overline{C}_{44}^1\eta_4 + \overline{C}_{44}^3\eta_4^3 + \overline{C}_{44}^5\eta_4^5 \tag{24}$$

where

$$\overline{C}_{44}^1\eta_4 = \frac{C_{44}^1}{A_{44} + I_4},\ \overline{C}_{44}^3\eta_4 = \frac{C_{44}^3}{A_{44} + I_4} \text{ and } \overline{C}_{44}^5\eta_4 = \frac{C_{44}^5}{A_{44} + I_4} \tag{25}$$

Bhattacharryya (1978) discussed nonlinear restoring moment by expressing it as an odd order polynomial of roll angle. Considering third order polynomial in roll, nonlinear restoring was simulated where the coefficients $\overline{C}_{44}^{1}$ and $\overline{C}_{44}^{3}$ were obtained from the approximation of the righting moment curve from an equation of the form:

$$\overline{GZ}(\eta_4) = (\overline{C}_{44}^{1}\eta_4 + \overline{C}_{44}^{3}\eta_{44}^{3}) \tag{26}$$

In general, the stability curve or the $\overline{GZ}$ curve can be obtained from the physical experiment. On fitting this curve with the polynomial described in (26), one can determine the corresponding coefficients. In the absence of such physical experiment, the representation of restoring moment becomes difficult. Nevertheless, attempts have been made for such representation through a series of numerical experiment to supplement experimental result. To emulate the spring action, one can assign suitable values of $\overline{C}_{44}^{1}$ and $\overline{C}_{44}^{3}$corresponding to particular type of vessel, based on two primary characteristics: (i) hard spring $C_{44}^{i} > 0$, and (ii) soft spring $C_{44}^{i} < 0$, i=3, 5, Hooft (1982). The coefficients of restoring moment may be obtained from the approximation of righting-moment curve using the polynomial approximation, Bhattacharryya (1978). Wright and Marsfield (1980), Feat and Jones (1981) and Bass (1982) included all the terms in the polynomial of the restoring moment where as Cardo et al. (1980-1984) considered only the linear and cubic terms. An important aspect of studying nonlinear restoring is to determine the influence of the initial conditions. Often such nonlinearity may lead to flare-up condition owing to the indirect inputs caused by the interactions between different motion components in higher degree of freedom.

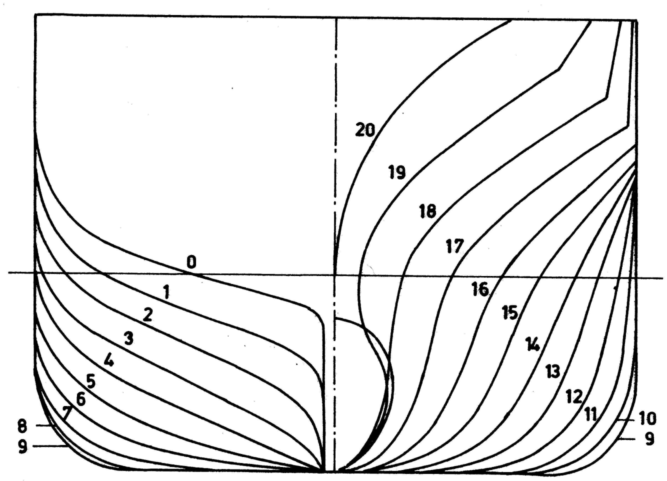

Figure 2: Body plan of a Panamax Container ship

For numerical experiment, computations are performed in time-domain for a Panamax Container ship under the action of sinusoidal wave of periodicity 11.2 sec with varying wave heights acting beam to the ship hull, when the ship is either stationary or having constant forward speed (U). The main particulars of the Container ship and body plan (Wang, 2000) are given in Figure 2. The sectional coefficients for added mass, damping, Froude-Krylov force and diffraction force corresponding to the wave period 11.2 sec are computed from the experimental results of Vugts (1968) and Frank's close-fit method (1970). This has been shown in Table 1. To start computation, the initial time step for numerical integration is specified as 0.1 sec.

Table 1: 2D-sectional values

Wave		Sectional coefficients							
Frequency Rad/sec ω	Period sec t	Sway added mass a_{22}	Roll added mass a_{44}	Sway-roll added mass a_{24}	Sway damping b_{22}	Roll damping b_{44}	Sway-roll damping b_{24}	Sway exciting force $f_2 + h_2$	Roll exciting moment $f_4 + h_4$
0.56	11.2	1.6	0.07	-.25	0.6	0.01	-0.07	2.25	1.9

4. Model Results and Discussion

From the review of the earlier literatures, it is observed that most of the researchers dealt with uncoupled roll motion with linear or nonlinear damping, and cubic or quintic representation of nonlinear restoring. These researchers obtained the restoring moment coefficients for cubic and quintic terms from the approximation technique of the righting moment curve.

Wright and Marshfield (1980) obtained the restoring coefficients for cubic ($\overline{C}_{44}^{3}$) and quintic ($\overline{C}_{44}^{5}$) terms, which are –0.3265 and 0.0114 respectively for high freeboard. In the computation of nonlinear ship roll damping, *Bass and Haddara (1988)* assumed the value of $\overline{C}_{44}^{3}$ as –1.45. *Cardo et al. (1980)* examined the influences of damping effects on the ship rolling motion in regular beam seas. He expressed the righting moment up to third order polynomial and obtained the value of $\overline{C}_{44}^{3}$ as –0.5495, which gives best fit for righting arm curve when the ship is fully loaded. Cardo et al. (1981) examined the nonlinear resonance for rolling of a ship in beam seas where the encounter frequency is an integer multiple or a sub-multiple of the natural frequency of the system. They solved the governing equation for two different situations, where $\overline{C}_{44}^{3}$= -1.75 and $\overline{C}_{44}^{3}$= 4, and found that for $\overline{C}_{44}^{3}$< 0, the corresponding righting arm curve at first reaches a relative maximum and then goes to zero, where as for $\overline{C}_{44}^{3}$>0, the curve increases monotonically in the whole range. This indicates that for $\overline{C}_{44}^{3}$>0, the system is not stable.

Table 2: Initial values corresponding to wave heights

Wave Height (m)	Initial conditions at $t = t_0$ sec						Remarks	
	Sway		Roll		Yaw		U=0 knots	U=10 knots
	η_2	$\dot{\eta}_2$	η_4	$\dot{\eta}_4$	η_6	$\dot{\eta}_6$		
1	2.0	0.0	1.222	0.0	0.15	0.0	FU	MC
2	2.0	0.0	0.8	0.0	0.15	0.0	FU	MC
3	2.0	0.0	0.1	0.0	0.15	0.0	FU	MC
4	2.0	0.0	0.001	0.0	0.15	0.0	FU	FU

To model the flared up conditions, the values of $\overline{C}_{44}^{3}$ and $\overline{C}_{44}^{5}$ must be assigned properly. After having numerical simulation for various initial conditions as mentioned in Table 2, the numerical values for $\overline{C}_{44}^{3}$ and $\overline{C}_{44}^{5}$ are obtained as -0.238 and -0.09 respectively. This corresponds to the behaviour of a soft spring. The performance of the ship with and without forward speed, and increase of wave height are shown in Table 2. In this table, 'FU' and 'MC' represent the abbreviated form for 'flared-up' and 'motion continue' respectively. If the motion is flared-up, this indicates that the system is unstable. Since the roll motion is having restoring property, the initial values pertaining to sway and yaw have no influence in the coupled condition. Hence, the roll initial conditions and the values of $\overline{C}_{44}^{3}$ and $\overline{C}_{44}^{5}$ are having great influence on motion time-histories.

FU = motion flared-up, MC = motion continue

To illustrate non-linear roll restoring while coupled motions are considered, several runs were taken by varying environmental conditions, ship speed and initial conditions (IC). The results shown in these figures were obtained from simulation studies for wave heights 1m, 2m, 3m and 4m with initial roll angles $\eta_4\big|_{t=0}$ = 0.001, 0.1, 0.8 and 1.222 degrees. We specify the initial conditions of sway and yaw corresponding to *Das and Das (2006a).* Figs. 3(a)-3(c) exhibit the comparison of harmonic behaviour of sway, roll and yaw while initial roll angle is set to 0.1. With the increase of wave height from 1m to 2m, motion amplitudes increase, preserving their periodicities. However, further increase of wave height beyond the regime of small amplitude harmonic response ($\geq$3m), oscillations become unstable. Such flaring up of harmonic response is caused due to the propagation of non-zero initial condition. The corresponding roll angle is found to be $\pm$1 degree without attenuation (Figure 3b).

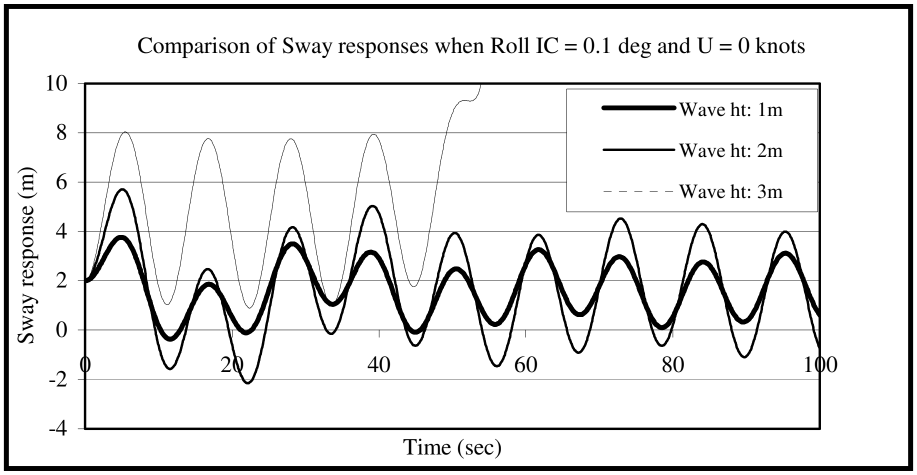

Figure 3: (a) Sway Response

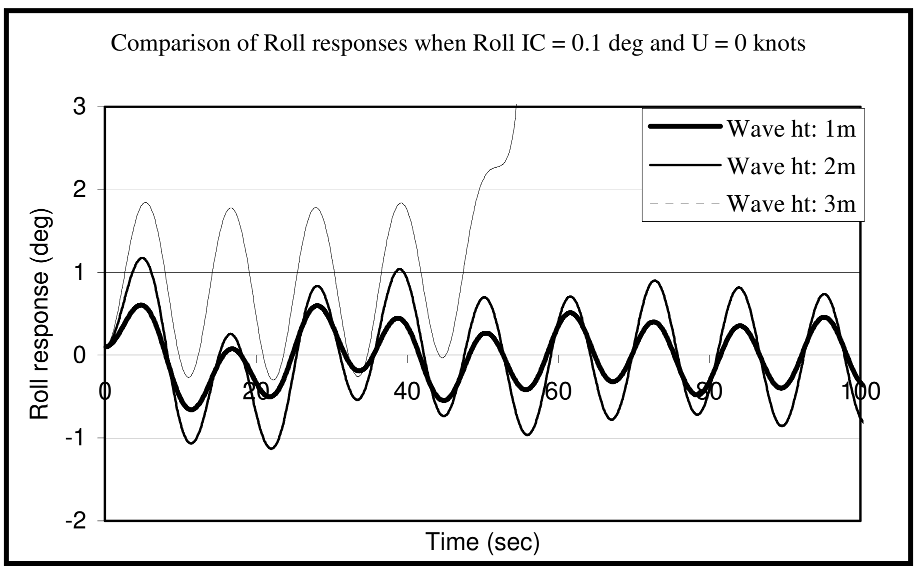

Figure 3: (b) Roll Response

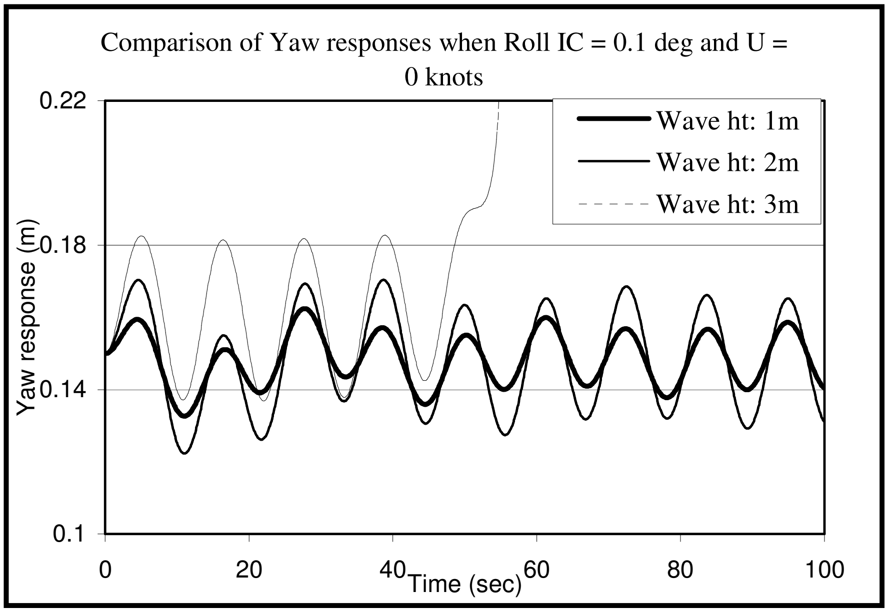

Figure 3: (c) Yaw Response

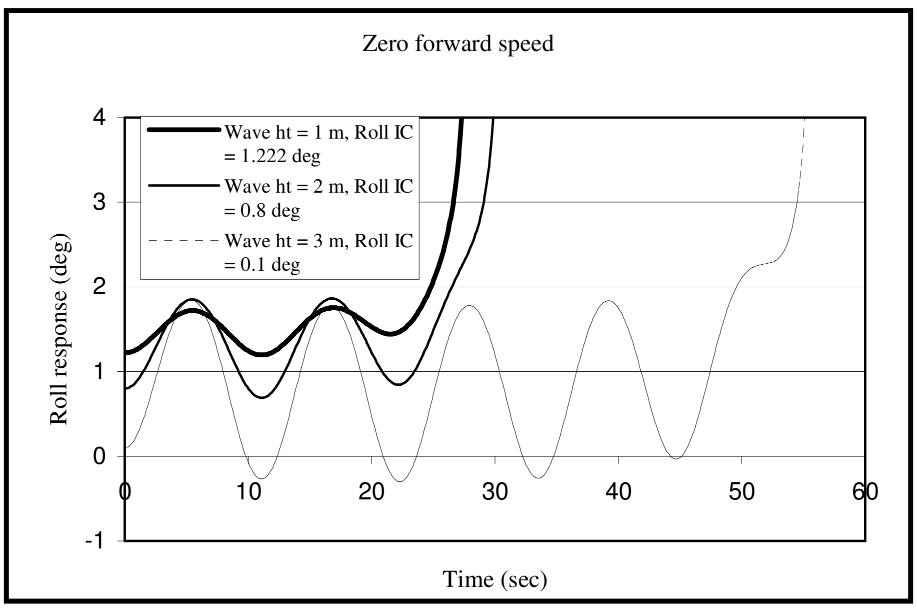

Figure 4: Comparison of roll responses due to different wave heights and initial conditions

The sway and yaw response also show the divergence in numerical solution obtained in Figs. 3(a) and 3(c). To understand relative contribution of initial disturbance and wave height, simulations were carried for various combinations of wave height and initial condition and three typical cases are illustrated here for comparison; (i) 1m wave height and IC $= \pm 1.222$ degree (ii) 2m wave height and IC $= \pm 0.8$ degree and (iii) 3m wave height and IC= ± 0.1 degree when forward speed is absent (Figure 4). These critical parameters form wave height-IC-speed matrix from which motion stability can be obtained for a particular wave frequency. The attenuation of roll amplitude and thereby control of roll motion for all time is noticed as the speed of the ship is increased from 0 knots to 10 knots (Figure 5). We analyze relative contribution of linear, cubic and fifth order terms of roll restoring moment $C_{44}^{r}(t)$ as specified in equation (11), and these are exhibited in Figs. 6(a)-6(c) for wave height, IC and speed variations. In the case of zero forward speed, ship fails to restore and the order of roll restoring moment increases from O (10^5) Newton-meter to higher order indicating oscillatory divergence. However, this oscillation is controlled due to constant forward speed (10 knots) or equivalently higher values for Froude number (within sub-critical range). Figure 7 shows the total roll restoring for three typical cases for comparison where the combined effect of linear, cubic and fifth order term behaves like a soft spring. Further increase of wave height, the roll motion is unbounded leading to capsize of ship even with small initial disturbance, IC= 0.001 deg, wave height is 4m and forward speed is 10 knots (Figure 8).

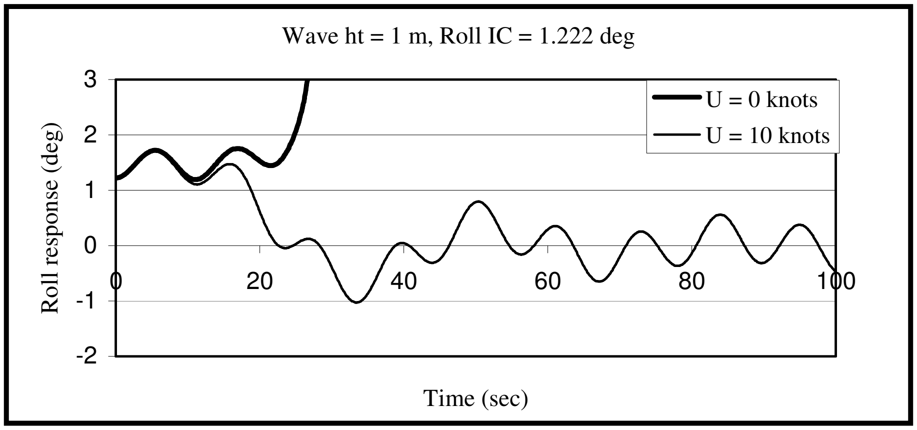

Figure 5: Comparison of roll responses when the ship is either stationary or having 10 knots speed

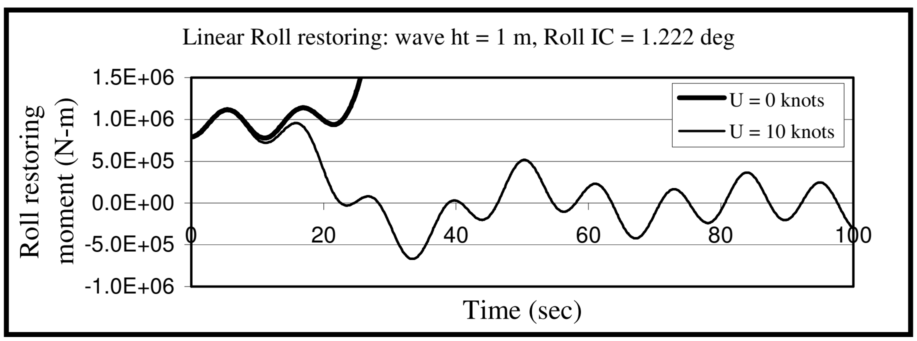

Figure 6: (a) Comparison of roll restoring: Linear order of roll for wave heights and IC

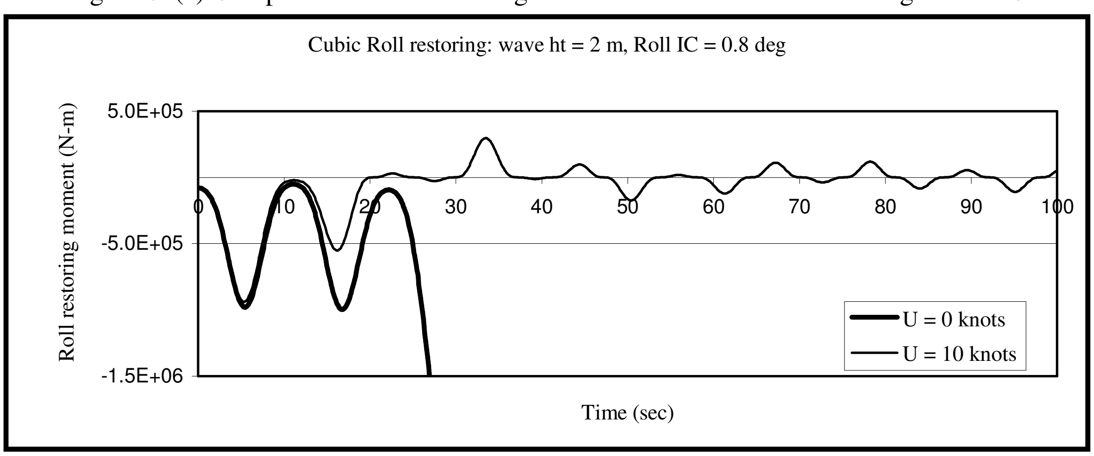

Figure 6: (b) Comparison of roll restoring: Cubic order of roll for wave heights and IC

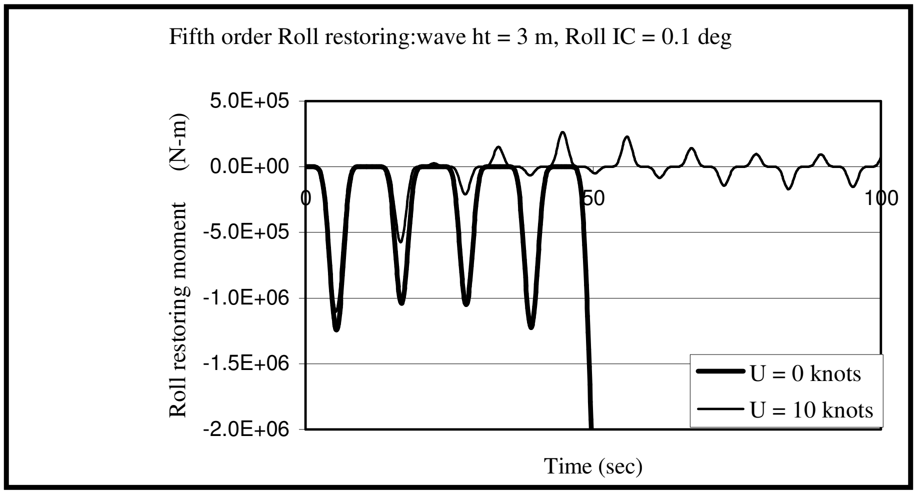

Figure 6: (c) Comparison of roll restoring: Fifth order of roll for wave heights and IC

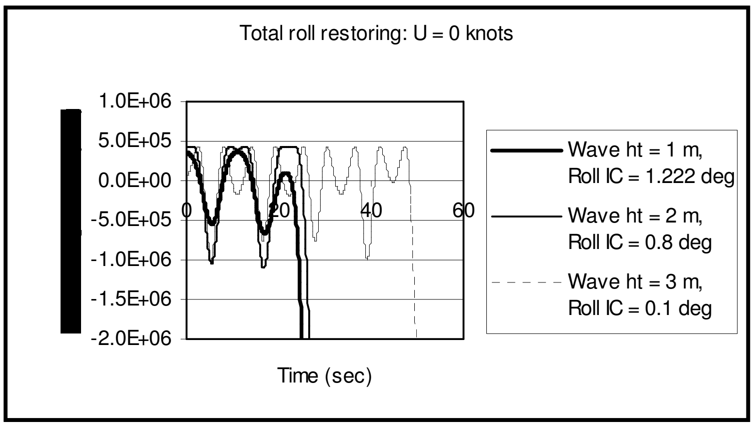

Figure 7: Comparison of total roll restoring

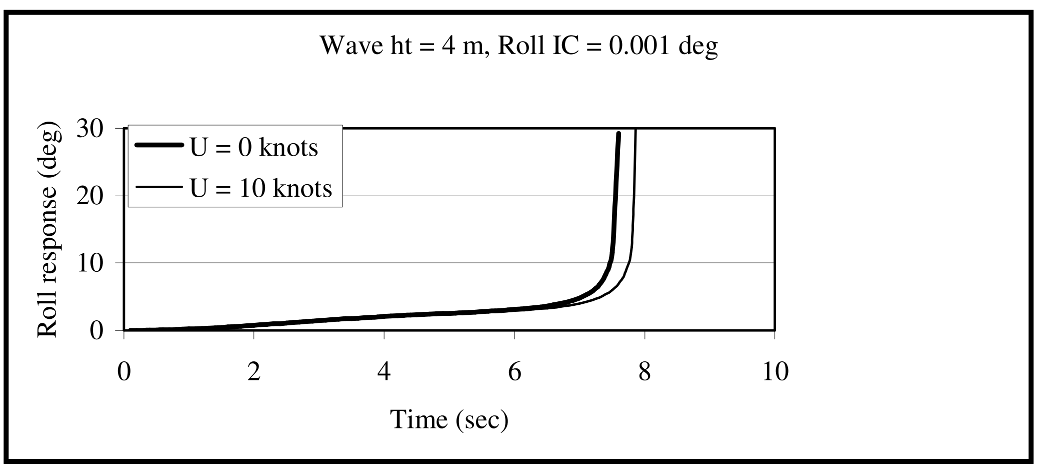

Figure 8: Comparison of roll responses for 4 m wave height and IC = 0.001 deg

5. Conclusion

This modelling approach described in this paper provides an important guideline to understand the role of parameters for stabilizing undesired roll oscillations and restoring mechanism. The damping moment considered in this paper is linear in form, however can be expressed as nonlinear to account viscous damping and variations in the mass moment of inertia. The important findings of this study elucidate that forward speed controls the motion oscillation and dampens the initial disturbance.

6. References

Bass, D.W. (1982): "On the response of biased ships in large amplitude waves", *International Shipbuilding Progress,* Vol. 29, pp 2-9

Bass, D.W. and Haddara, M.R. (1988): "Nonlinear models of ship roll damping" *International Shipbuilding Progress,* Vol. 35, pp 5-24

Bhattacharyya, R. (1978): "Dynamics of Marine Vehicles*", John Willey & Sons*, New York

Cardo, A., Ceschia, M., Francescutto, A. and Nabergoj, R. (1980): "Effects of the angle-dependent damping on the rolling motion of ships in regular beam seas", *International Shipbuilding Progress,* Vol. 27, pp135-138

Cardo, A., Francescutto, A. and Nabergoj, R. (1981): "Ultraharmonics and subharmonics in the rolling motion of a ship: steady-state solution", *International Shipbuilding Progress,* Vol. 28, pp 234-251

Cardo, A., Francescutto, A. and Nabergoj, R. (1982): "On damping models in free and forced rolling motion", *Ocean Engineering,,* Vol. 9, pp 171-179

Cardo, A., Francescutto, A. and Nabergoj, R. (1984): "Nonlinear rolling response in a regular seas", *International Shipbuilding Progress,* Vol. 31, pp 204-208

Cummins, W.E. (1962): "The impulse response function and ship motions", *Schiffstechnik.* 9, pp101-109.

Das, S. N. and Das, S. K. (2004): "Determination of Coupled Sway, Roll and Yaw Motion of a Floating body in Regular Waves", *International Journal of Mathematics and Mathematical Sciences,* Vol. 41, pp 2181- 2197

Das, S. N. and Das, S. K. (2005): "Mathematical Model for Coupled Roll and Yaw Motions of a Floating Body in Regular Waves under Resonant and Non-resonant Conditions", *Applied Mathematical Modelling,* Vol. 29, pp 19-34

Das, S. K. and Das, S. N. (2006a): "Modelling and analysis of coupled nonlinear oscillations of a floating body in two degrees of freedom", *Acta Mechanica,* Vol. 181, pp 31-42

Das, S. N., Sahoo, P.K. and Das, S. K. (2006b): "Determination of roll motion for a floating body in regular waves", *Proceedings of the IMechE Part M: Journal of Engineering for the Maritime Environment,* (in press)

Dalzell, J.F. (1978): "A note on the form of ship roll damping", *Journal of Ship Research,* Vol. 22, pp 178-185

Feat, G. and Jones, D. (1981): "Parametric excitation and the stability of a ship subjected to steady heeling moment", *International Shipbuilding Progress,* Vol. 28, pp 263-267

Frank, W. and Salvesen, N. (1970): "The Frank Close-Fit Ship-Motion Computer Program", *Report 3289, NSRDC*, Washington, D.C.

Haddara, M.R. (1973): “On nonlinear rolling of ships in random seas”, *International Shipbuilding Progress,* Vol. 20, pp 377-387

Hooft, J.P. (1982): “Advanced Dynamics of Marine Structures”, *John Willey & Sons*, New York.

Nayfeh, A.H. and Khdeir, A.A. (1986): “Nonlinear rolling of ships in regular beam seas”, *International Shipbuilding Progress,* Vol. 33, pp 40-49

Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flanuery, B.P. (1992): “Numerical Recipes in Fortran 77”, *Cambridge University Press*, New York

Roberts, J.B. (1984): “Comparison between simulation results and theoretical predictions for a ship rolling in random beam waves”, *International Shipbuilding Progress,* Vol. 31, pp 168-180

Salvesen, N., Tuck, E.O. and Faltinsen, O.M. (1970): “Ship motions and sea loads”, *Trans. Society of Naval Architects and Marine Engineer,* Vol. 78, pp 250-287

Tick, L.J. (1959): “Differential equations with frequency-dependent coefficients”, *Journal of Ship Research*, Vol. 3, pp 45-46

Virgin, L.N. (1987): “The nonlinear rolling response of a vessel including chaotic motions leading to capsize in regular seas”, *Applied Ocean Research*, Vol. 9, pp 89-95

Vugts, J.H. (1968): “The hydrodynamic coefficients for swaying, heaving and rolling cylinders in a free surface”, *Report 194, Shipbuilding Lab., Delft University of Technology*, Delft

Wright, J.H.G. and Marshfield, W.B. (1980): “Ship roll response and capsize behaviour in beam seas”, *Trans. Royal Institute of Naval Architects*, Vol. 122, pp 129-148

Wang, Z. H. (2000): “Hydroelastic Analysis of High-Speed Ship”, *PhD Thesis, Dept Naval Architecture and Offshore Engineering, Technical University of Denmark,* Lyngby